Peripheral Stochasticity in Tokamaks. The Martin-Taylor Model Revisited

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We analyse the effect of an Ergodic Magnetic Limiter on the magnetic field line dynamics in the edge of a large aspect-ratio Tokamak. We model the limiter action as an impulsive perturbation and use a peaked-current model for the Tokamak equilibrium field. The theoretical analysis is made through the use of invariant flux functions describing magnetic surfaces. Results are compared with a numerical mapping of the field lines.

1. Introduction

One of the various proposed techniques to reduce plasma-wall interactions in Tokamaks is the Ergodic Magnetic Limiter (EML) concept [1, 2]. The basic mechanism of EML action is the creation of a boundary layer of ergodic magnetic field *, in order to reduce the heat loading at the Tokamak inner wall. Recent experiments have shown a decrease of impurity level in the plasma core due to EML action [3].

The simpler implementation for an EML consists of a ring-shaped arrangement of m pairs of conductors wound around the Tokamak (see Figure 1). Martin and Taylor [4] were able to study magnetic field ergodization in this EML proposal by means of a poloidal Poincaré mapping, but they have employed a model restricted to the Tokamak edge region, for both equilibrium and EML fields. Nevertheless, they point to the right direction, when considering the peripheral magnetic surface destruction as the main source of field ergodization.

This achievement has led us to revisit their model, using a systematic method for magnetic surface description. Notice that some models for the Tokamak equilibrium field were combined with numerical evaluation of field line trajectories caused by EML action [5], but few analytical results were found up to now. We try to circumvent the extremely complicated nature of EML field, treating the limiter action as an impulsive perturbation. The mapping of field lines is also simplified in this approach, giving analytical results even when toroidal effects are considered.

* The words *ergodic*, *stochastic* and *chaotic* will be used as meaning actually the same thing.

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2. Equilibrium and Limiter Field

In this paper we assume a large aspect ratio geometry by using cylindrical coordinates, as depicted in Figure 1. The Tokamak equilibrium field in this approximation can be written as $\mathbf{B}^{(0)} = (0, B_\theta(r), B_0)$, where $B_0 = \text{const}$ and the poloidal field $B_\theta(r)$ is calcu-

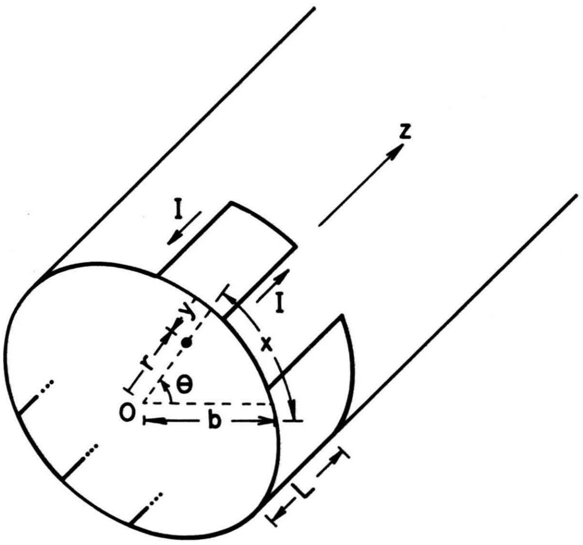

Fig. 1. Geometry of an Ergodic Magnetic Limiter.

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lated through the use of the peaked current profile:

$$\mathbf{j}(r) = j_0 \left(1 - \frac{r^2}{a^2}\right)^{\gamma} \hat{z}\,, \tag{1}$$

where a is the plasma column radius and $\gamma > 0$.

The magnetic field lines lie over magnetic surfaces characterized by an invariant flux function Ψ_0 satisfying $\mathbf{B}^{(0)} \cdot \nabla \Psi_0 = 0$. In the case of fields with helical symmetry, one finds such an invariant as

$$\Psi_0(r, u) = m A_z(r, u) + \frac{n}{R_0} r A_\theta(r, u)\,, \tag{2}$$

where $m(n)$ is the poloidal (toroidal) mode number, R_0 the Tokamak major radius, $\mathbf{A}$ the vector potential and $u = m\theta - \frac{nz}{R_0}$. Equation (2) leads to a particular form for an equilibrium flux function, which reads [6]

$$\Psi_0 = \frac{n B_0 r^2}{2 R_0} - m \int^{r} \mathrm{d}r' B_\theta(r')\,. \tag{3}$$

The safety factor which characterizes these surfaces is written in the form $q(r) = \frac{R_0 B_\theta(r)}{r B_0}$.

The limiter field consists mainly on the contributions given by the toroidally oriented wires (m pairs of length L, see Fig. 1) conducting a current I in opposite directions for adjacent conductors. In a first approximation we neglect the finiteness of the wires. The irrotational EML field is conveniently described by means of a scalar potential expanded in a harmonic series. However, for $r \approx b$ (b is the Tokamak minor radius) this field can be approximated by a single harmonic (for a detailed derivation see [7]) with the flux function

$$\Psi_{10}(r, \theta) = \frac{\mu_0 m I}{\pi} \left(\frac{r}{b}\right)^m \cos(m\theta)\,. \tag{4}$$

Although the helical dependence is absent in this expression, it will be recovered when considering the finite extension of the limiter wires. This can be accomplished by using an impulsive periodic excitation*:

$$\Psi_1(r, \theta, z) = \Psi_{10}(r, \theta)\, L \sum_{j=-\infty}^{+\infty} \delta(z - 2\pi R_0 j)\,. \tag{5}$$

Using the Poisson sum formula

$$\sum_{j=\infty}^{+\infty} \delta(k-j) = 1 + 2 \sum_{n=1}^{+\infty} \cos(2\pi k n)$$

we may pick up the resonances created by this kind of configuration, namely when $\mathrm{d}u = m\,\mathrm{d}\theta - \frac{n}{R_0}\mathrm{d}z \approx 0$:

$$(\Psi_1)_{m/n}(r, u) = \frac{\mu_0 m I L}{2\pi^2 R_0} \left(\frac{r}{b}\right)^m \cos u\,, \tag{6}$$

which exhibits helical dependence, as expected.

3. Destruction of Magnetic Islands

A given resonance, whose helicity is described by the mode numbers (m, n) generates m magnetic islands roughly around an unperturbed rational surface with safety factor $q(r_{m/n}) = m/n$. In order to compute the islands' widths we superpose the equilibrium and limiter fields:

$$\Psi_{m/n}(r, u) = \Psi_0(r) + (\Psi_1)_{m/n}(r, u)\,. \tag{7}$$

Using (3) and (6), and expanding this combination around $r_{m/n}$, the half-width of a magnetic island caused by EML action is

$$\Delta r_{m/n} = \left[\frac{2\mu_0 m I L}{\pi^2 R_0 \Psi_0''(r_{m/n})} \left(\frac{r_{m/n}}{b}\right)^m\right]^{1/2}. \tag{8}$$

The proposal of Karger, Lackner, and Feneberg [1, 2] indicates island overlapping in the Tokamak peripheral region as the basic mechanism for generation of stochasticity. This region comprises the outer portion of the plasma column plus the vacuum scrape-off layer between the material limiter and the vessel wall.

The Chirikov overlapping criterion is very useful in order to estimate threshold currents for peripheral ergodization. If we consider two neighbour resonances, namely with mode numbers m/n and m'/n', the criterion leads to the following condition (including the two-thirds prescription):

$$S = \frac{\Delta r_{m/n} + \Delta r_{m'/n'}}{|r_{m/n} - r_{m'/n'}|} \geq \frac{2}{3}\,, \tag{9}$$

where S is the so-called stochasticity parameter. A possible choice of neighbouring islands is the pair (8/1, 8/2), according to an MHD equilibrium in which the safety factor at the plasma edge is equal to 5.0 (corresponding to $\gamma = 4.0$).

In Fig. 2 the stochasticity parameter for this couple of islands is plotted against the normalized limiter current. Parameters are taken from the TBR-1 small

* A Fourier analysis of a realistic square-pulse EML action would give similar results for $L \ll R_0$ and small values of n.

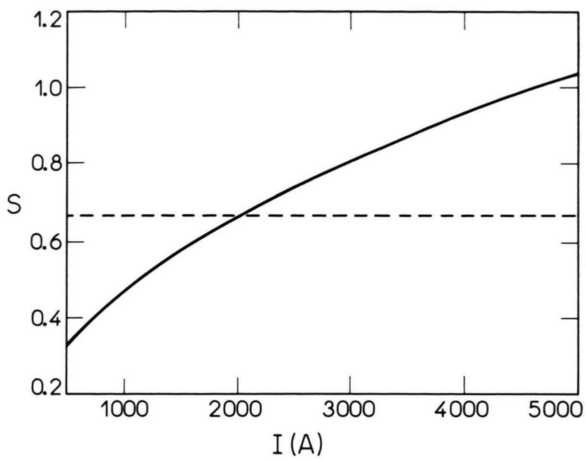

Fig. 2. Stochasticity parameter as a function of Limiter current for 8/1 – 8/2 island overlapping, using TBR-1 parameters and $L = 0.08$ m. The dashed line indicates the 2/3 threshold.

Tokamak operating at the University of São Paulo ($a = 0.08$ m, $b = 0.11$ m, $R_0 = 0.30$ m, $B_0 = 0.32$ T, $I_p = 10.0$ kA). The Chirikov condition (8) is verified only for values of the EML current ($\approx 20\%$ of the plasma current) one order of magnitude larger than typical values for helical windings [6]. This result indicates the need of several ergodic limiters in order to obtain current values closer to those that have been used in helical limiters. This is a point that has been noted in other works on the subject [8].

4. Magnetic Field Line Mappings

The use of an impulse model for EML action enables us to give simple analytical forms to the Poincaré-type puncture plots of the magnetic field lines. The procedure, although approximate, uses less computer time than previous codes based in ab initio Biot-Savart evaluations of the EML field [5]. The equilibrium and limiter fields are taken from the formulae of the preceding section. We treat the impulsive perturbation by means of a simple procedure for this class of problems [9], defining discretized variables for the radial as well as angular positions of the points on the mapping:

$$\begin{aligned} r_n &= \lim_{\varepsilon \to 0} r(z = 2\pi R_0 n + \varepsilon), \\ r_n^* &= \lim_{\varepsilon \to 0} r(z = 2\pi R_0 (n+1) - \varepsilon), \\ r_{n+1} &= \lim_{\varepsilon \to 0} r(z = 2\pi R_0 (n+1) + \varepsilon) \end{aligned} \tag{10}$$

and similar definitions for θ_n, θ_n^*, θ_{n+1}. These are coordinates of successive piercings of a given field line on a surface of a section located at $\phi = \text{const}$.

The map reads

$$r_{n+1} = r_n^* - \xi \left(\frac{r_n^*}{b}\right)^{m-1} \sin(m\theta_n^*), \tag{11a}$$

$$\theta_{n+1} = \theta_n^* - \xi b \left(\frac{r_n^*}{b}\right)^{m-2} \cos(m\theta_n^*), \tag{11b}$$

where

$$\xi = \frac{\mu_0 m I L}{B_0 \pi}, \tag{12}$$

and

$$r_n^* = r_n, \tag{13a}$$

$$\theta_n^* = \theta_n + \frac{2\pi B_\theta(r_n) R_0}{r_n B_0}. \tag{13b}$$

In (13 b) the second term is the rotational transform of the field line.

We can also include the so-called toroidality effect on the toroidal field B_ϕ, by taking

$$B_\phi = \frac{B_0}{1 - \dfrac{r}{R_0}\cos\theta}, \tag{14}$$

so that we rewrite (13 b) as

$$\theta_n^* = 2\arctan[\lambda^{-1}(r_n)\tan(\Omega(r_n) + \arctan \Xi(r_n, \theta_n))] + 2\pi, \tag{15}$$

where we have defined

$$\Omega(r_n) = \frac{\pi R_0 B_\theta(r_n)(1 - \varepsilon(r_n))}{B_0 r_n \lambda(r_n)}, \tag{16}$$

$$\lambda(r_n) = \frac{1 - \varepsilon(r_n)}{\sqrt{1 - \varepsilon^2(r_n)}}, \tag{17}$$

$$\Xi(r_n) = \tan\left(\frac{\theta_n}{2}\right)\lambda(r_n), \tag{18}$$

$$\varepsilon(r_n) = \frac{r_n}{R_0}. \tag{19}$$

5. Conclusions

Two approaches were used in order to describe the magnetic field line topology caused by EML action on a large aspect-ratio Tokamak. Firstly, we introduce an invariant flux function to characterize perturbed magnetic surfaces of the equilibrium field. The resonances between the EML and Tokamak fields show up as islands whose dimensions were estimated. Each island is surrounded by a thin layer of stochastic motion, which is enlarged when neighbouring islands approach mutually. A simple criterion to evaluate threshold external currents for peripheral ergodization gives a value quite large when compared to other magnetic divertors. It is better to regard it as an upper bound, rather than an exact value.

The second way to analyse the field line flow is the Poincaré surface of section technique. The impulsive character of the EML action, as supposed in our model, enables us to readily obtain an analytical form for the resulting mapping. A detailed analysis of the dynamical features of this mapping, as well as numerical examples are being completed and will be published elsewhere.

Acknowledgements

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